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Variable-*G* Cosmology and Creation

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It has recently been asserted that a universe with a time-varying gravitational "constant" G necessarily implies creation if the rest mass of matter particles is constant. It is shown that this is not necessarily true. An example of a cosmological model with variable G and Λ is presented, in which there is no creation and in which the rest mass of matter particles is constant.

1. INTRODUCTION

In a recent article Alfonso-Faus (1986) has proposed that a universe with a time-varying gravitational "constant" G necessarily implies creation if the rest mass of matter particles m_p is constant. It is furthermore asserted that, apart from the assumed law $G \propto t^{-1}$ (Dirac, 1937), the other consequences of the large numbers hypothesis do not hold. Einstein's field equations are assumed by Alfonso-Faus, except that G is permitted to be a function of time, and a Robertson–Walker metric is taken to apply.

We show in this note that variable G does not necessarily imply creation if the rest mass of matter particles is constant. Furthermore, we point out that an example of a Friedmann-type universe with varying G and no creation, consistent with Dirac's large numbers hypothesis, already exists in the literature (Lau, 1985). Finally, we point out that the assumption that G varies at t^{-1} does not seem to be borne out by observations.

Throughout, we adopt the view, also taken by Alfonso-Faus, that there is only one set of units, the same for microphysics as well as macrophysics. It is known (Dirac, 1982) that if we allow different sets of units, then variable-G cosmology need not necessarily imply creation.

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2. VARIABLE G VERSUS CREATION

Alfonso-Faus begins with the field equations in the form

$$R_{ab}-\tfrac{1}{2}Rg_{ab}=GT_{ab} \tag{1}$$

where G is permitted to be a function of time. We are using units in which the speed of light is unity and the constant 8π has been absorbed into G. For the sign conventions we are using, we refer to Ellis (1971).

From the form of equation (1), we note first that a zero cosmological term has been assumed. However, the most general form of the left side of equation (1) is not $R_{ab}-\tfrac{1}{2}Rg_{ab}$, but $R_{ab}-\tfrac{1}{2}Rg_{ab}+\Lambda g_{ab}$, where Λ is a real constant (Cartan, 1922), which nowadays is identified with the cosmological constant. There are numerous other reasons for believing in a nonzero Λ. The cosmological constant has been considered by various authors within the context of quantum field theories, quantum gravity, supergravity theories, Kaluza-Klein theories, the inflationary universe scenario, particle physics, and grand unified theories [see references cited in Singh and Singh (1983), Lorentz-Petzold (1984), and Banerjee and Banerjee (1985)]. In fact, Λ is today regarded as the vacuum energy energy density of the quantum field (Zeldovich, 1968) and it is believed that Λ is related to the mass of the Higgs boson.

Second, since Alfonso-Faus abandons the idea of a constant G, there is no compelling reason for a constant Λ. Rather, there are several reasons for advocating a variable Λ. Although relatively small at present, it is widely believed that Λ was large during the early stages of the universe and strongly influenced its expansion. It has been suggested that Λ depends on the Higgs scalar field (Bergmann, 1968; Wagoner 1970). Linde (1974) proposed that Λ is a function of temperature and related it to the process of broken symmetries. Other important implications for the early inverse have been discussed (e.g., Kasper, 1985; Villi, 1985; DerSarkissian, 1985).

Motivated by the above, we start with the field equations in the form

$$R_{ab}-\tfrac{1}{2}Rg_{ab}+\Lambda g_{ab}=GT_{ab} \tag{2}$$

where both Λ and G are permitted to be functions of time. Spatial homogeneity and isotropy lead to the Robertson-Walker metric

$$ds^2=-dt^2+R^2(t)[dr^2/(1-kr^2)+r^2(d\theta^2+\sin^2\theta\, d\phi^2)]$$

The perfect fluid form for the energy momentum tensor is

$$T_{ab}=(\mu+p)u_a u_b+pg_{ab} \tag{3}$$

where μ is the energy density of the fluid, p the pressure, and $u^a=\delta_0^a$ the fluid four-velocity.

By making use of the Robertson–Walker metric and the form (3) of the energy momentum tensor, we obtain from the field equations (2) the following equation

$$\dot{\mu} + \mu \dot{G}/G + 3(\mu + p)\dot{R}/R + \dot{\Lambda} = 0 \tag{4}$$

where the overdot denotes a derivative with respect to time. Equation (4) is the analog of the usual conservation of mass-energy equation

$$\dot{\mu} + 3(\mu + p)\dot{R}/R = 0 \tag{5}$$

which is obtained by taking the divergence of equation (3). It follows from equations (4) and (5) that we may have the usual energy conservation law holding also in the case of variable Λ and G providing that

$$\mu \dot{G}/G = -\dot{\Lambda} \tag{6}$$

A viable Friedmann model if this kind, consistent with mass-energy conservation and Dirac's large numbers hypothesis, has been constructed by Lau (1985). For convenience we reproduce below some of the main features of the model:

$$G \propto t^{-1} \tag{7}$$

$$\Lambda \propto t^{-2} \tag{8}$$

$$R \propto t^{1/3} \tag{9}$$

$$\mu \propto t^{-1} \tag{10}$$

It may readily be seen that there is no creation in this model. Since the mass within a comoving volume $V \propto R^3$ is given by $N_p m_p = \mu R^3$, where N_p is the number density of particles and m_p the particle mass, we see from relations (9) and (10) that

$$\frac{d}{dt}(N_p m_p) = 0$$

From this, it is apparent that if m_p is kept constant, N_p is also constant, and there is no creation.

Thus, variable-G cosmology does not necessarily imply creation. It is only if the cosmological term in the field equations vanishes that we have creation. Finally, we point out that observations seem to indicate that a variation of G of the kind $G \propto t^{-1}$ proposed originally by Dirac (1937) does not seem likely (Hellings et al., 1983; Krasinsky et al., 1985). Hence, there is a need for extension of both models discussed, if possible, to other variations of G, such as $G \propto t^n$, and we are presently investigating such possibilities.

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